UNIVERSITY OF MANITOBA | Department of Electrical and Computer Engineering

24.781 Computational Electromagnetics

ASSIGNMENT 2
Solution of Time-Domain Problems by Finite Difference Methods

October 26, 2005 **Due Date: Thursday, November 3, 2005**

A) The transmission line equations are given as

$$\begin{cases} \frac{\partial}{\partial x}v(x, t) + L\frac{\partial}{\partial t}i(x, t) + Ri(x, t) = 0 \\ \frac{\partial}{\partial x}i(x, t) + C\frac{\partial}{\partial t}v(x, t) + Gv(x, t) = 0 \end{cases}$$

for a line having series per-unit-length (p.u.l.) resistance, R [Ω/m]; series p.u.l. inductance, L [H/m]; shunt p.u.l. capacitance, C [F/m]; and shunt p.u.l. conductance, G [S/m]. For an infinite homogeneous line (L, C, R, G constants), derive the solution for the voltage, $v(x, t)$, and current, $i(x, t)$, along the line at any time $t > 0$ given initial conditions $v(x, 0) = v_0(x)$ and $i(x, 0) = i_0(x)$. Derive the solution for the two cases: *i*) lossless ($R = G = 0$), and *ii*) lossy ($R \neq 0$, $G \neq 0$).

B) Write a program to calculate the voltage and current along a 1 metre long transmission line ($0 \leq x \leq 1$) using the Yee version of the Leap-Frog scheme. (Comment all codes and include them as an appendix to your assignment.) Assume a lossless transmission line that has p.u.l. capacitance $C = 50$ pF/m and inductance $L = 0.5$ μH/m. Also assume that line is short-circuited at the $x = 0$ end and open-circuited at the $x = 1$ end. What is the velocity, c_0, of propagation for this line?

Assume that the initial conditions are $v_0(x) = 10(u(x - 0.45) - u(x + 0.55))$ where $u(x)$ is the unit step-function, and plot the voltage and current along the line at the following times: 2.2, 2.3, 2.5, 3.5 ns. Run your program for the time step equal to 0.9, 1.0, and 1.01 times the Courant limit $\Delta t = \Delta x / c_0$. (Choose whatever spatial discretization Δx you deem appropriate.) Compare to the exact solution on the same plot.

C) Modify your program to handle lossy lines. Assume the line of question B has 0.1 [Ω/m] of p.u.l. resistance and 0.001 [S/m] of p.u.l conductance. Run your code again at the Courant limit for each of the times given in question B and include plots of the exact solution only for time 2.2 ns.

D) Modify your program to deal with resistive terminations and run it again at the Courant limit for the lossless cases when the $x = 0$ end is terminated with 100 Ω and the $x = 1$ end is terminated with 50 Ω (No need to derive the exact solution.)